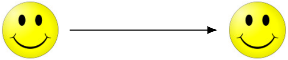
The Adversarial Noise Threshold for Distributed Protocols

William M. Hoza and Leonard J. Schulman
Caltech

January 10, 2016

Coding for interactive communication

- Classic coding theory: Alice sends a message to Bob



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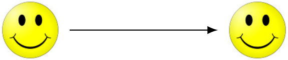

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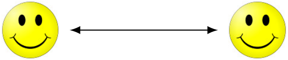

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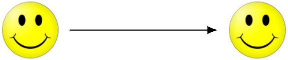

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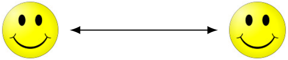

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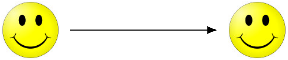

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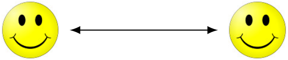


- E.g. chess over the phone
- "What?"

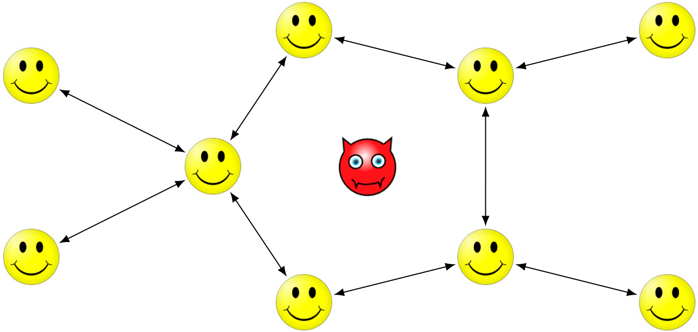
Topic of this talk

- Challenge: Protect distributed computation from channel noise
- Coding for interactive *multiparty* communication

Outline

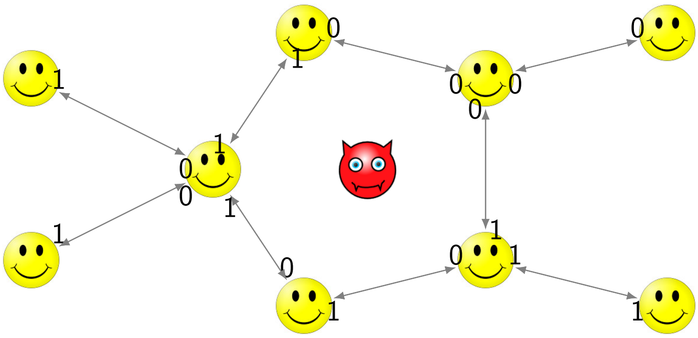
1. The model
2. Related work
3. Main result
4. Proof sketch
5. Directions for further research
6. Acknowledgements

The model



- Parties $P_1, \ldots, P_n$ connected by m two-way communication channels (arbitrary, static topology)
- Input to a computational problem split up as $x = (x_1, \ldots, x_n)$, with P_i receiving x_i

The model (2)



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- Adversary sees all, flips bits as she sees fit

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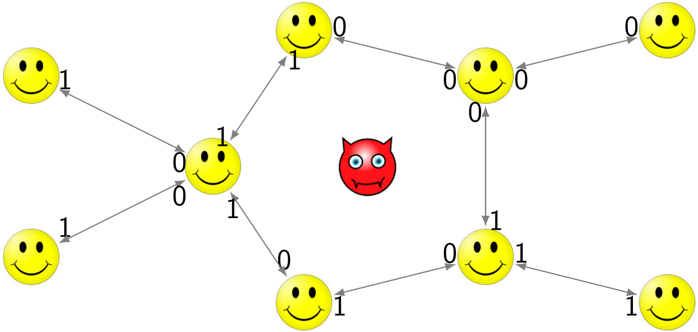

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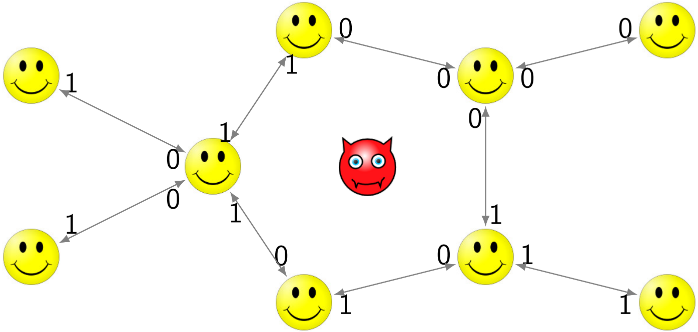

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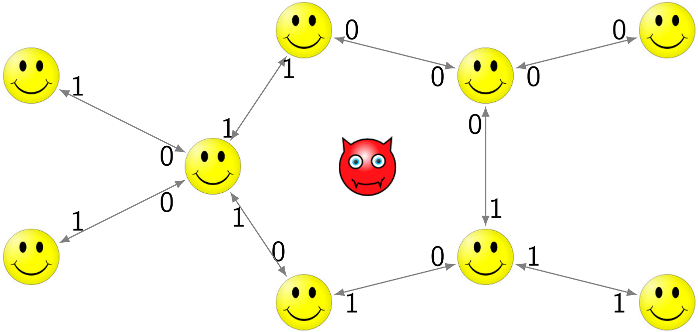

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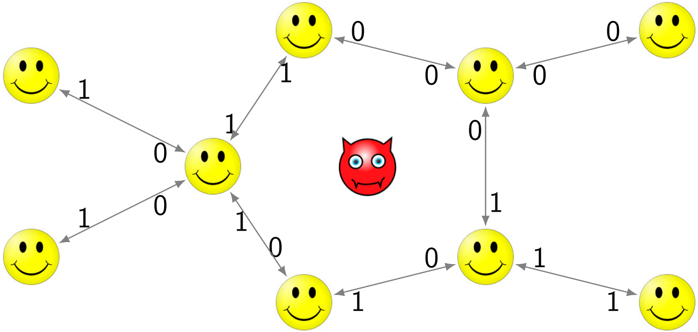

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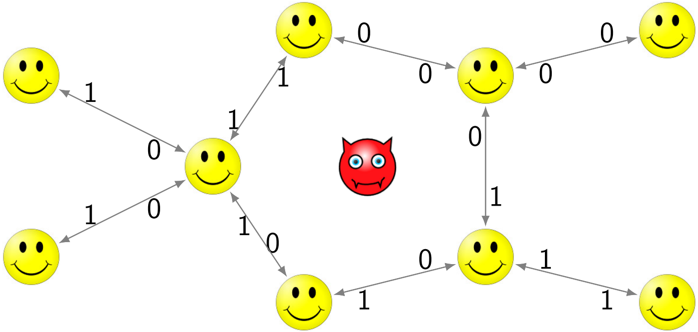

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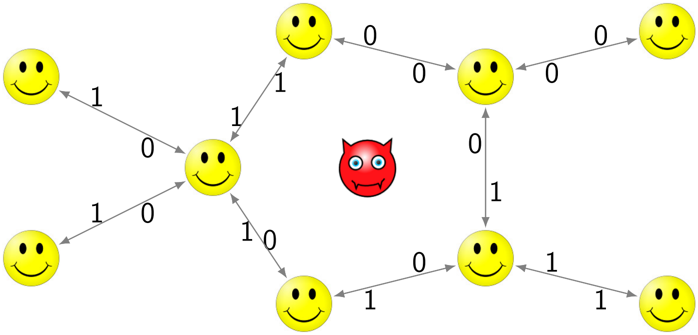

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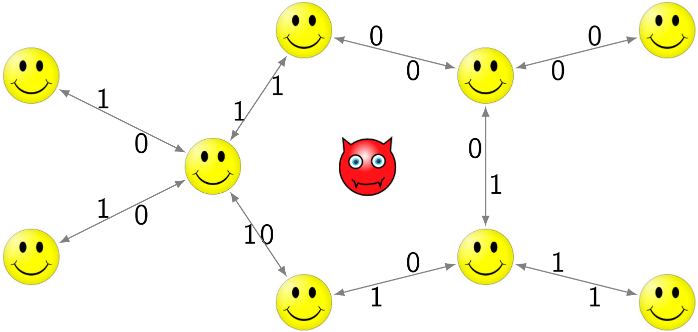

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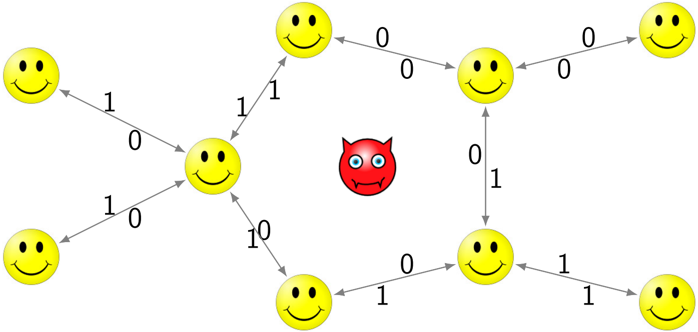

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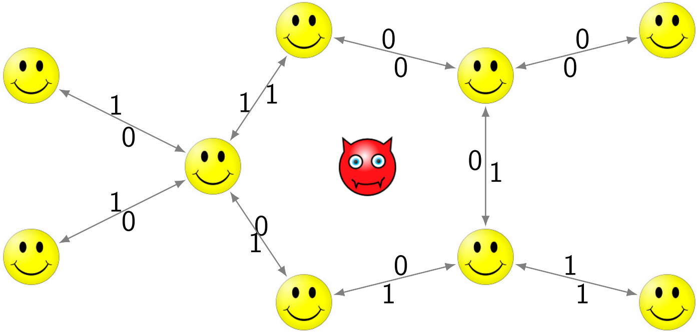

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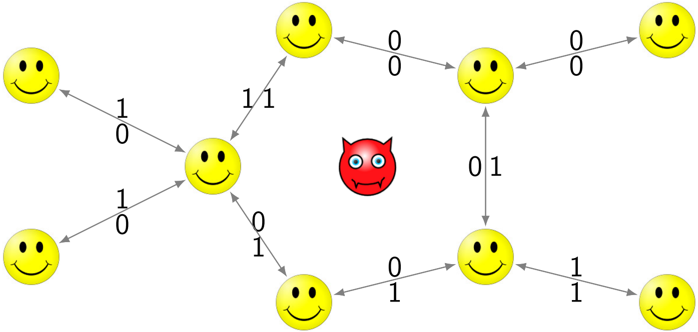

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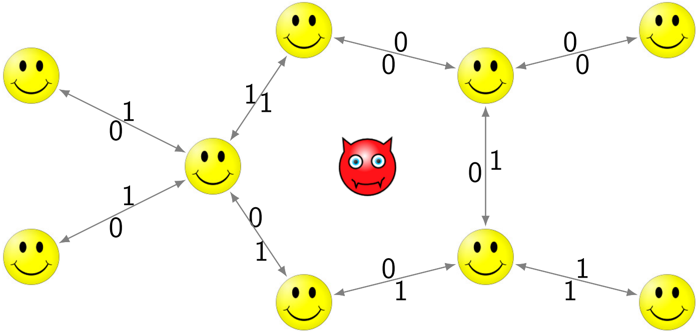

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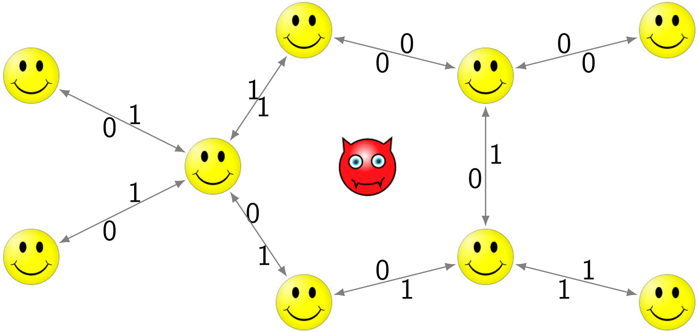

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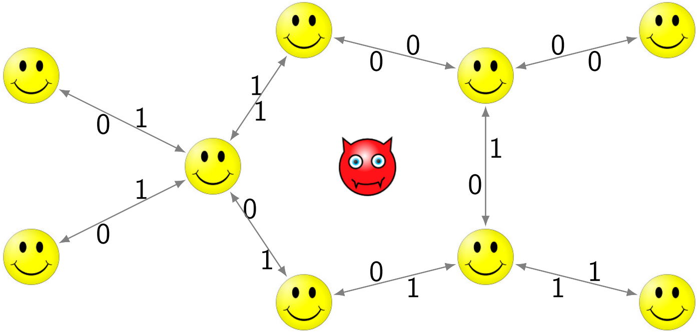

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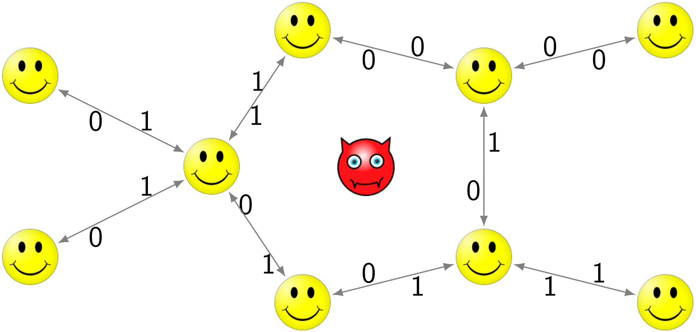

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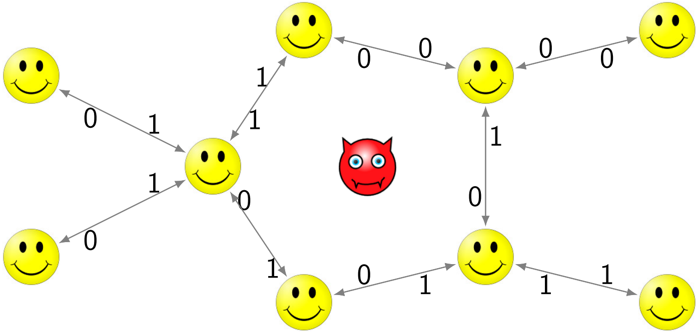

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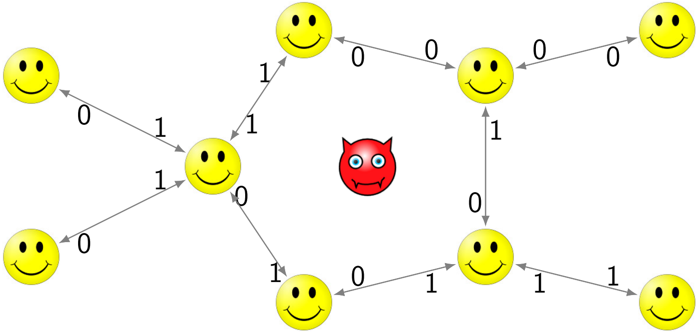

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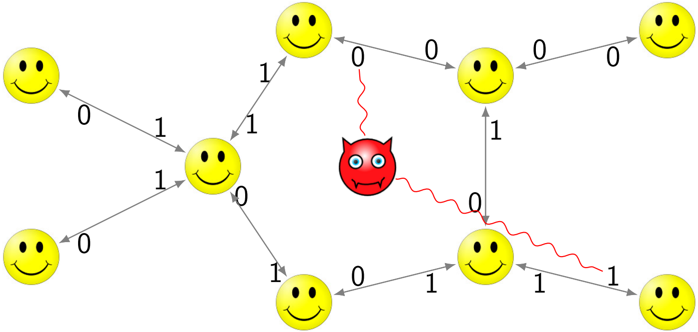

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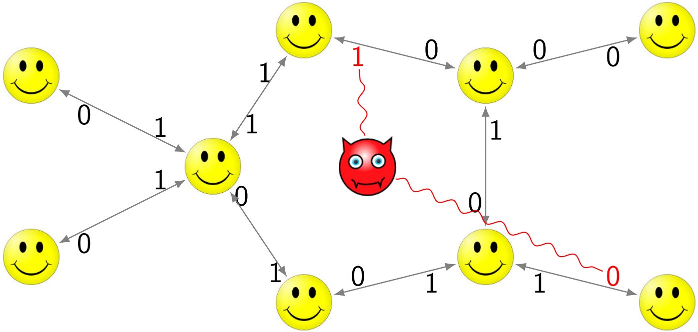

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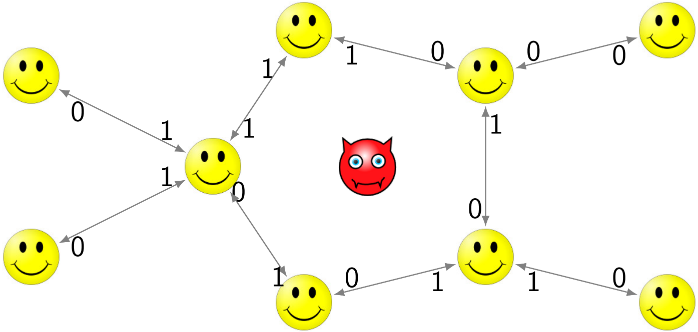

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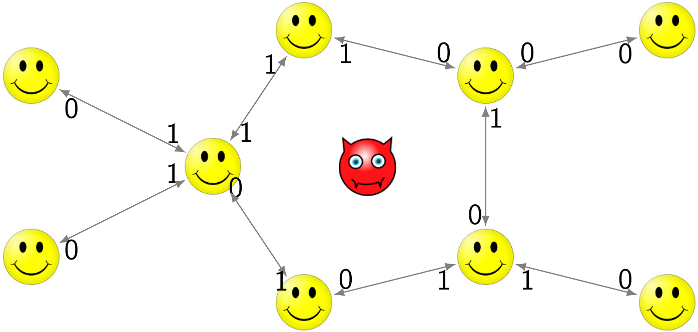

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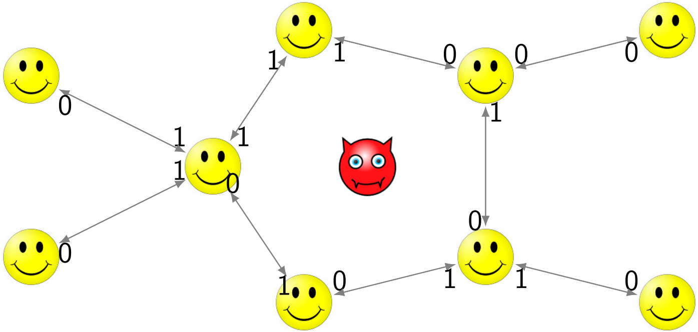

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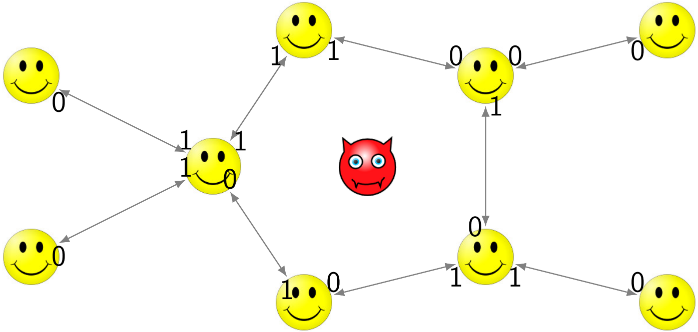

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The model (3)

- *Protocol*: n-tuple of (possibly probabilistic) algorithms that the parties use to decide what bits to transmit
- After T rounds, each party gives an output
- T is the *round complexity* of the protocol (known by all parties)

The model (4)

- Goal: Design *compiler* C: transforms protocol π into simulation protocol $\widetilde{\pi} = C(\pi)$ which tolerates a high error rate

$$\text{Error rate} = \frac{\text{Total number of bits flipped}}{\text{Total number of bits transmitted}}$$

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- Secondary goal: $\widetilde{\pi}$ should have low round complexity

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- See also Gelles, Moitra, Sahai '11, '14

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- Proof: Adversary attacks k edges to effectively disconnect graph

Figure: "Concentrate all your fire on the nearest starship."

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  - $k =$ edge connectivity of G

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- "RS compiler"

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- So final protocol tolerates error rate $\Omega(1/\widetilde{m}) = \Omega(1/n)$

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$$\frac{10m}{n} \left|\widetilde{\delta}(U)\right| \geq |\delta(U)|. \tag{1}$$

  - $\delta(U)$ is the set of edges in G crossing U
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- Builds on Batson et. al '09, Benczúr and Karger '96

Proof of main result: Step 1 (cont.)

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- Theorem (Leighton, Maggs, Rao '94): For any set $\mathcal{P}$ of simple paths, there is a schedule for sending one packet along each path in $\mathcal{P}$ in a total of $\mathcal{O}(\text{congestion} + \text{dilation})$ time steps.

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- Each round of π is simulated by $\mathcal{O}(\frac{m \log n}{n})$ steps in π' by using the paths of Lemma 2 $\square$

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