Near-Optimal Pseudorandom Generators for Constant-Depth Read-Once Formulas

Dean Doron[1]
UT Austin → Stanford

Pooya Hatami[2]
UT Austin → Ohio State

William M. Hoza[3]
UT Austin

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[3] Supported by the NSF GRFP under Grant DGE-1610403 and by a Harrington Fellowship from UT Austin

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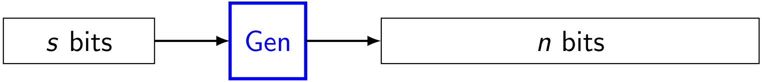

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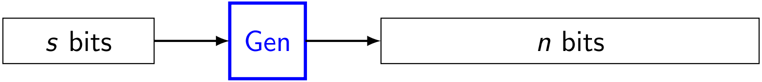

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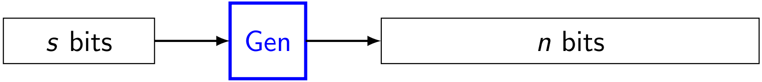

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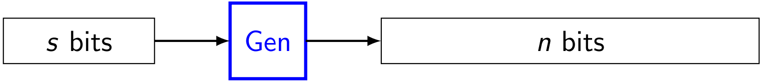

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- Minimize seed length $s = s(n, \varepsilon)$

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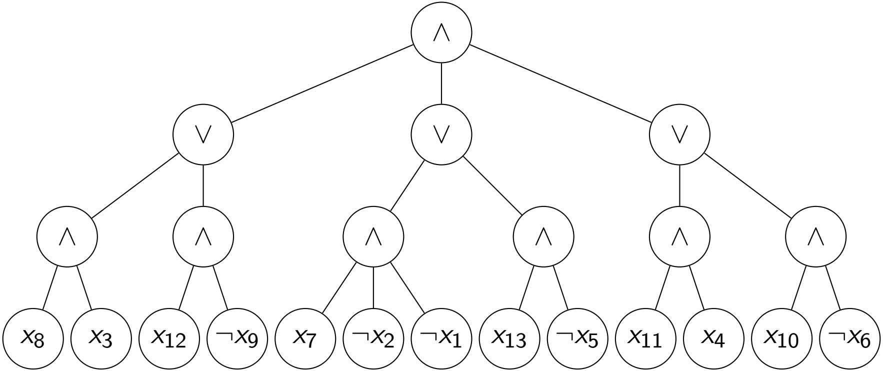

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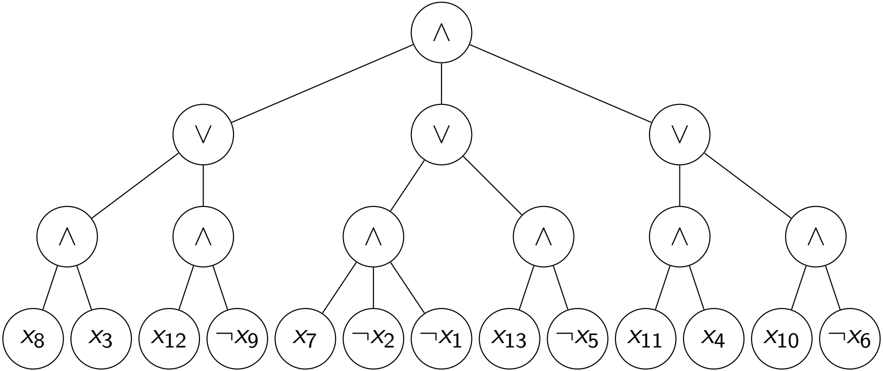

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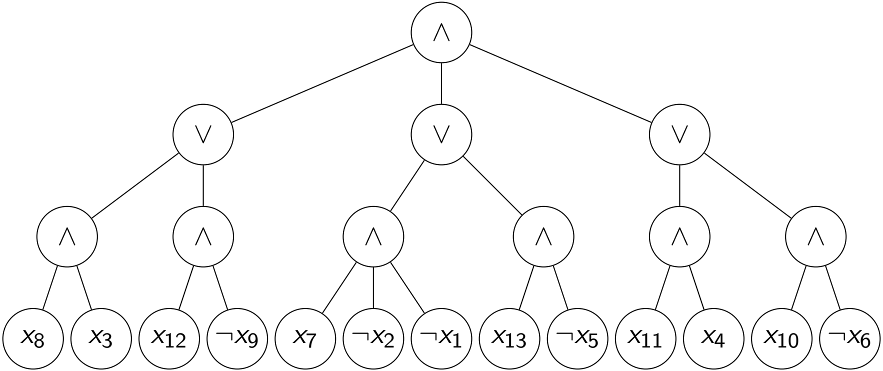

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- Read-once version of $\mathbf{AC}^0$

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- **Main result:** PRG for read-once $\mathbf{AC}^0$ with seed length

$$\log(n/\varepsilon) \cdot O(d \log\log(n/\varepsilon))^{2d+2}.$$

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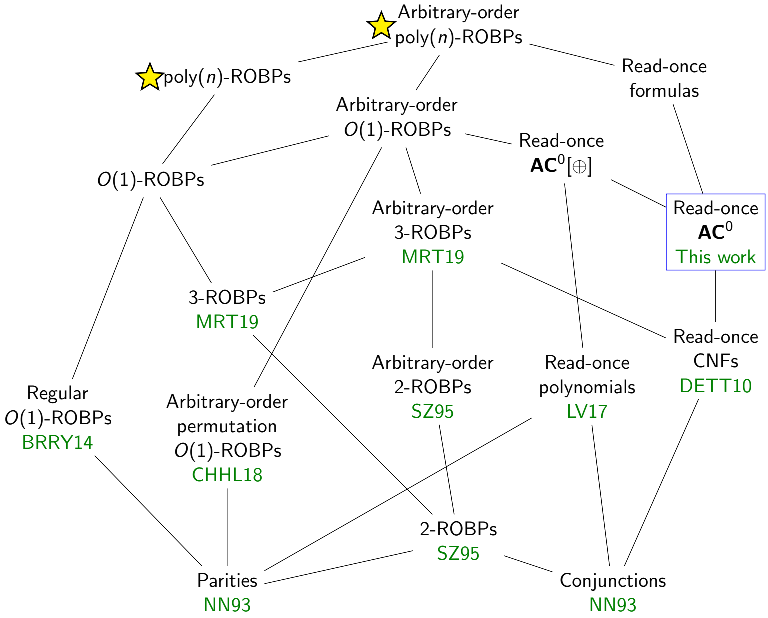
- Read-once $\mathbf{AC}^0$ is one of the frontiers of this progress

Seed length $\widetilde{O}(\log n)$



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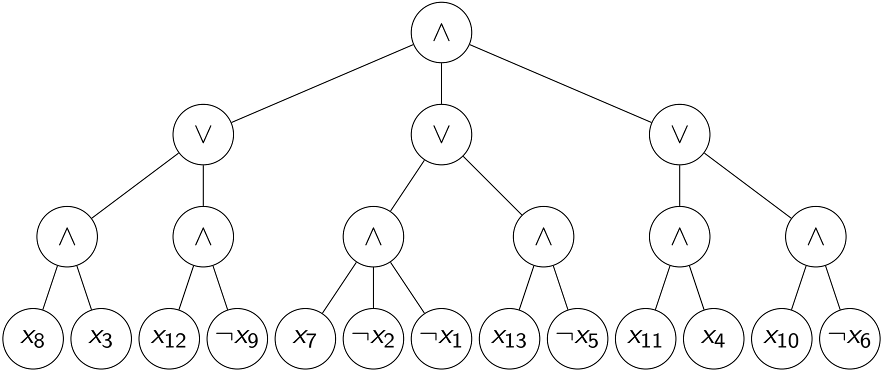
PRGs via pseudorandom restrictions [AW89]



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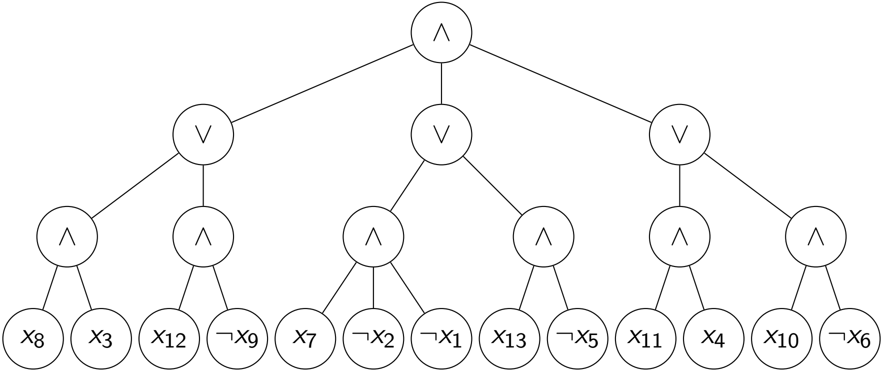

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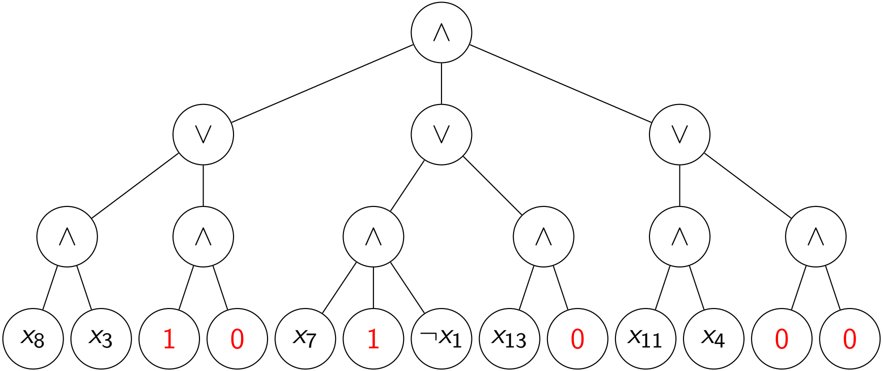

Restriction notation

- Define $\mathrm{Res}\colon \{0,1\}^n \times \{0,1\}^n \to \{0,1,\star\}^n$ by

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$y =$	0	1	1	0	0	1	0	0
$z =$	0	0	1	1	1	1	0	1
$\mathrm{Res}(y,z) =$	0	$\star$	$\star$	1	1	$\star$	0	1

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- A distribution D over $\{0,1\}^n$ is ε-biased if it fools parities:

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- (Proof involves clever Fourier analysis, building on [RSV13, HLV18, CHRT18])

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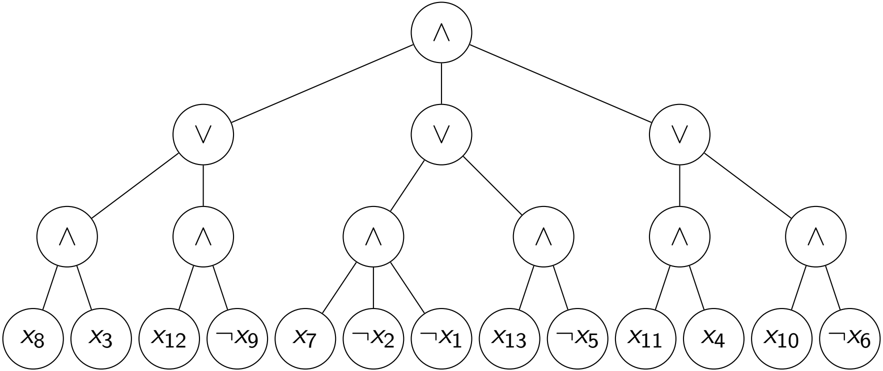

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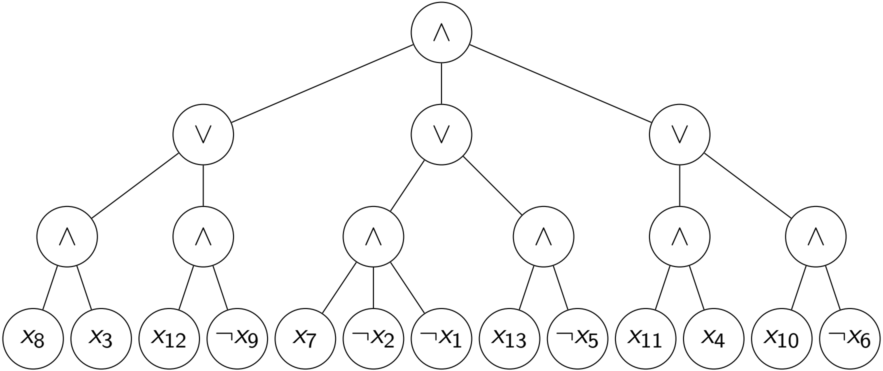

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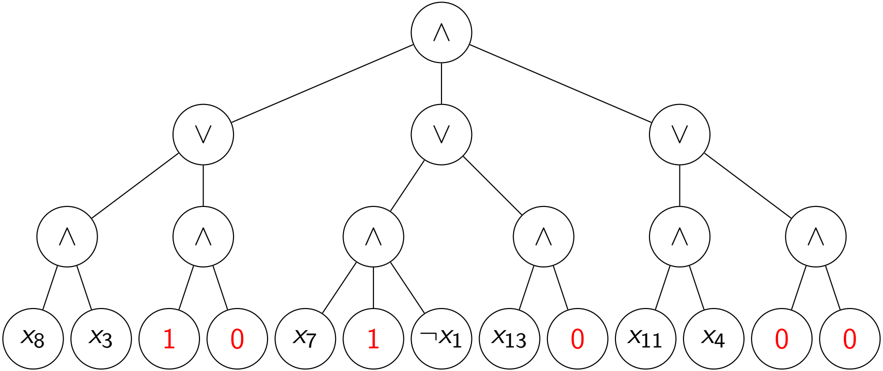

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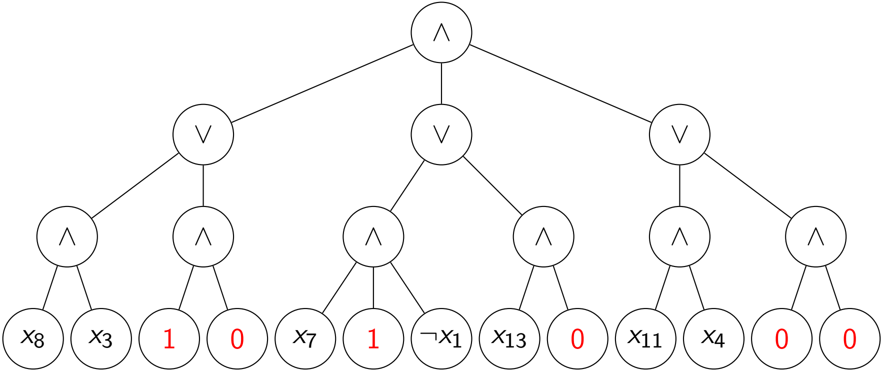

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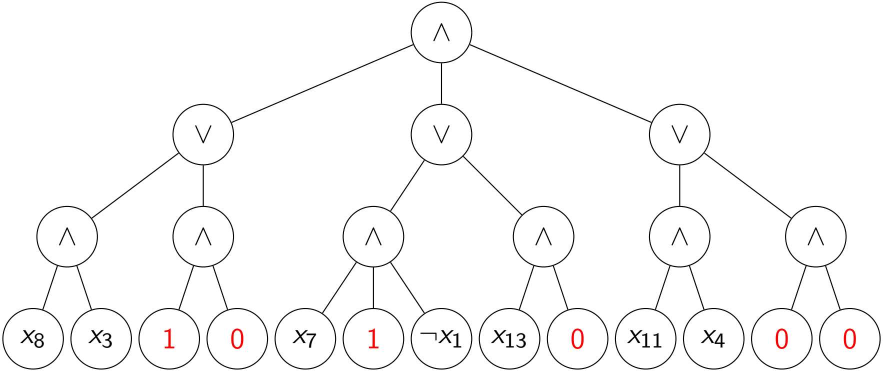

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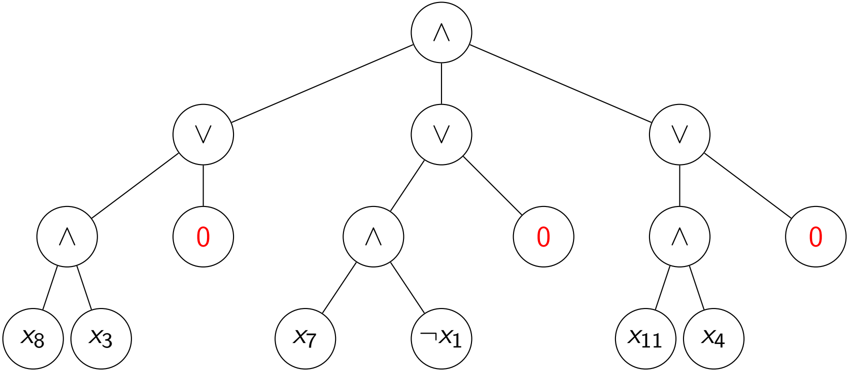

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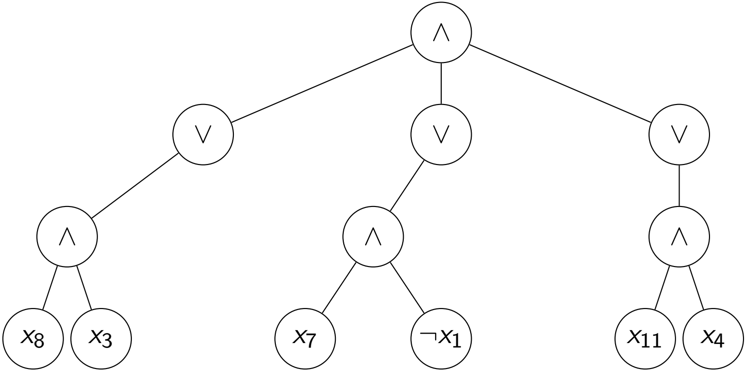

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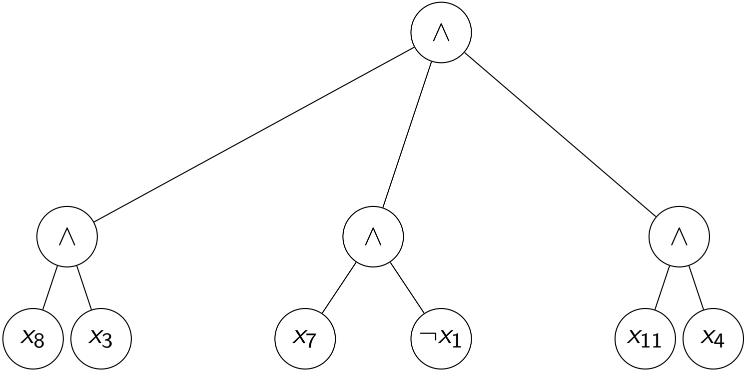

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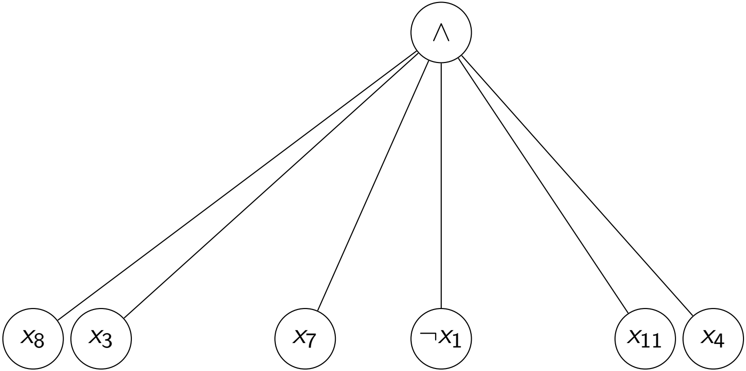

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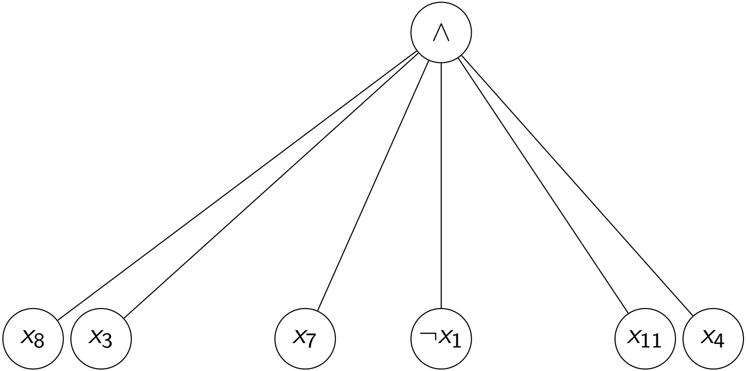

- Step 2: Fool restricted formula, taking advantage of simplicity

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3. $X = \mathrm{Res}(G_d \oplus D, G'_d \oplus D')$

Preserving expectation

- **Claim**: For any depth-$(d+1)$ read-once $\mathbf{AC}^0$ formula f,

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- **Proof**: Read-once $\mathbf{AC}^0$ can be simulated by constant-width ROBPs [CSV15]
- So we can simply apply Forbes-Kelley result:

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Simplification

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- **Main Lemma**: With high probability over $X^{\circ t}$,

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  where $t = O((\log \log n)^2)$
- Actually we only prove this statement "up to sandwiching"

$\Delta \mapsto \text{polylog}\, n$: Proof outline

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$\Delta \mapsto \mathrm{polylog}\, n$: Proof outline

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$\Delta \mapsto$ polylog n: Proof outline

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- Testing for simplification is another read-once $\mathbf{AC}^0$ problem
- So we can derandomize the [CSV15] analysis:

$$X = \text{Res}(G_d \oplus D, G'_d \oplus D')$$

Collapse under truly random restrictions

- Assume f is a biased read-once $\mathbf{AC}^0$ formula:

$$\mathbb{E}[f] \leq \rho \text{ or } \mathbb{E}[f] \geq 1 - \rho$$

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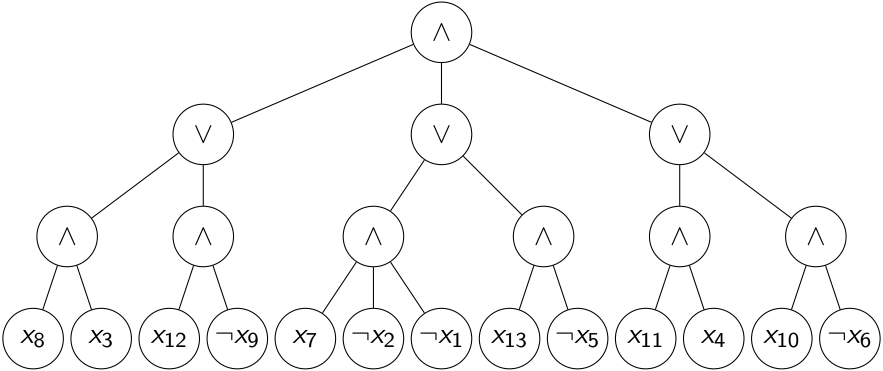
- (Proof uses Fourier analysis)

NAND formulas



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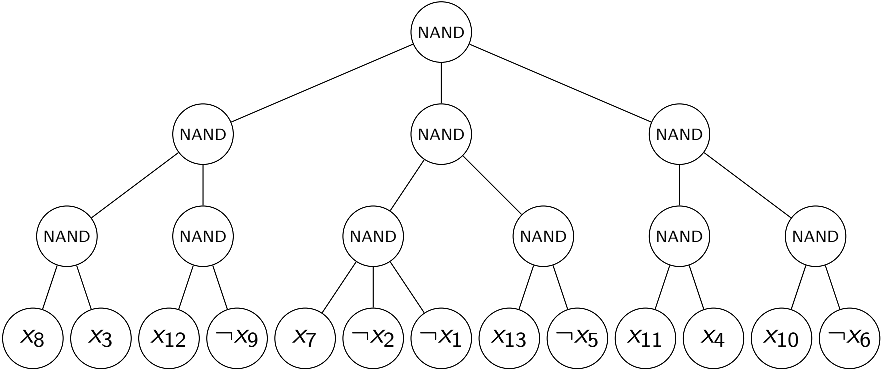

Collapse under truly random restrictions (continued)

- **Corollary**: If $\mathbb{E}[f] \geq 1 - \rho$, then

$$\Pr_{R^{\circ s}}[f|_{R^{\circ s}} \not\equiv 1] \leq 2\rho + \frac{1}{n^{100}}$$

Collapse under truly random restrictions (continued)

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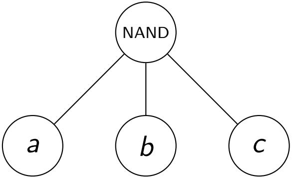
- **Lemma**: Can be decided in depth-d read-once $\mathbf{AC}^0$

Deciding whether $\exists f \in \mathcal{F}, f|_{\mathsf{Res}(y,z)} \equiv 1$



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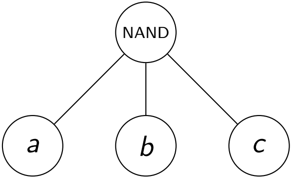

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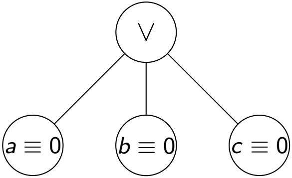

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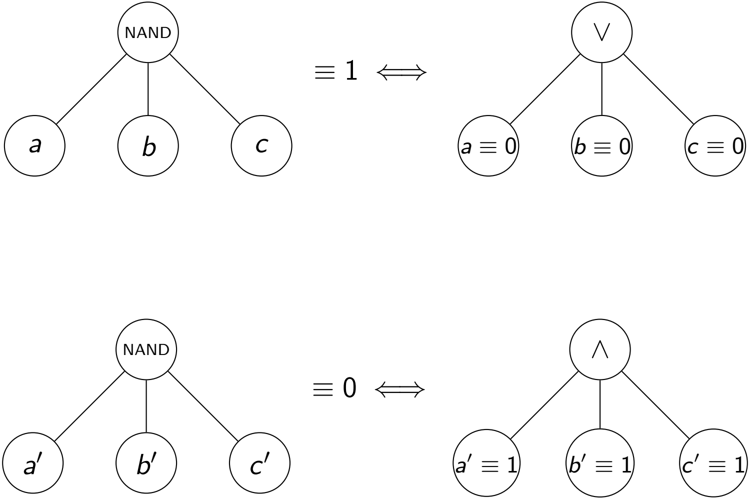

Deciding whether $\exists f \in \mathcal{F}, f|_{\mathrm{Res}(y,z)} \equiv 1$ (continued)

- At bottom, we get one additional layer:

$$(\mathrm{Res}(y,z)_i \equiv b) \iff (y_i = 0 \wedge z_i = b)$$
$$(\neg\,\mathrm{Res}(y,z)_i \equiv b) \iff (y_i = 0 \wedge z_i = 1 - b)$$

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- At top: "$\exists f \in \mathcal{F}$" is one more $\vee$ gate (merge with top $\vee$ gates)

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$$\Pr_X[\forall f \in \mathcal{F}, f|_X \not\equiv 1] \approx \Pr_R[\forall f \in \mathcal{F}, f|_R \not\equiv 1]$$

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$$\Pr_{X^{\circ s}}[\forall f \in \mathcal{F}, f|_{X^{\circ s}} \not\equiv 1] \approx \Pr_{R^{\circ s}}[\forall f \in \mathcal{F}, f|_{R^{\circ s}} \not\equiv 1]$$

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$$\leq \left(2\rho + \frac{1}{n^{100}}\right)^{|\mathcal{F}|}$$

$\Delta \mapsto \sqrt{\Delta}$ polylog n

- So far: $X^{\circ s}$ causes any biased depth-$(d-1)$ formula to collapse

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- **Lemma**: With high probability over $X^{\circ s}$,

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Illustration: $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$

Total depth $d+1$

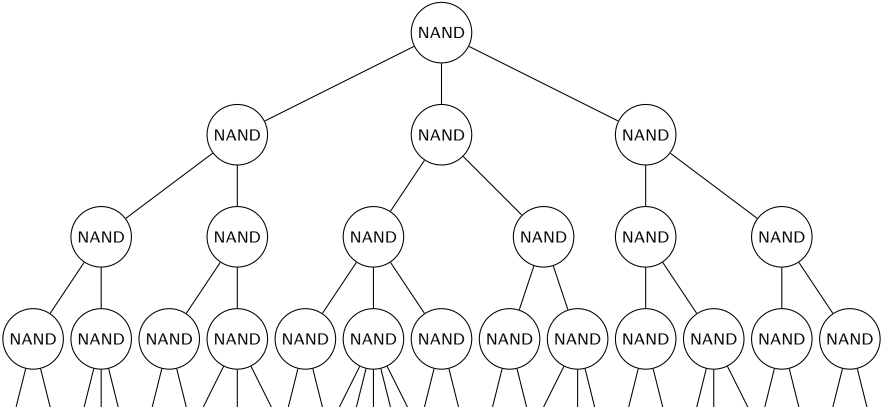
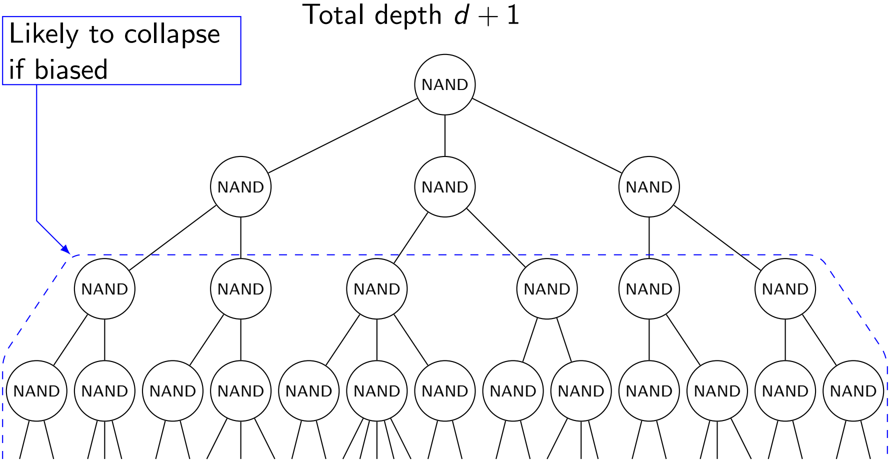


Illustration: $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$



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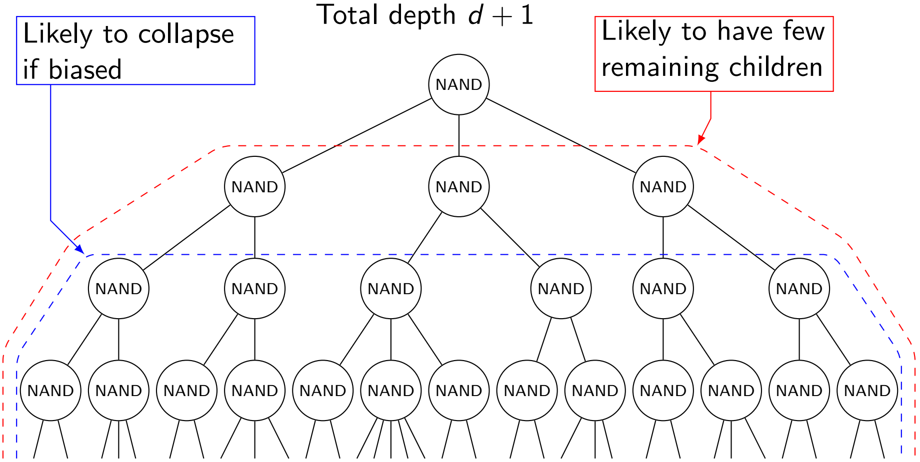

Proof that $\Delta \mapsto \sqrt{\Delta}\,\mathrm{polylog}\, n$

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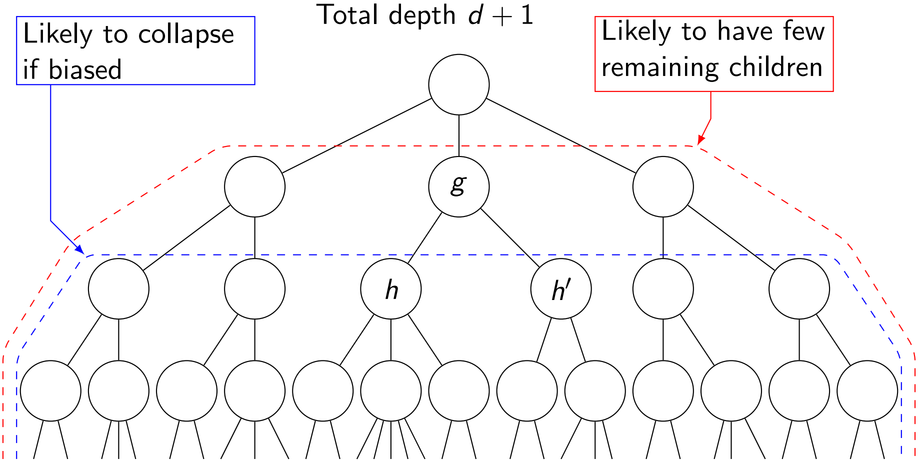
- Consider one bucket $\mathcal{B} = \{h : \mathbb{E}[h] \approx 1 - \rho\}$

Illustration: $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$ (continued)



Proof that $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$ (continued)

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Proof that $\Delta \mapsto \sqrt{\Delta}\,\mathrm{polylog}\, n$ (continued)

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Proof that $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$ (continued)

- $\mathcal{B} = \{h : \mathbb{E}[h] \approx 1 - \rho\}$
- How big can $\mathcal{B}$ be?
- $$1/\,\text{poly}(n) \leq \mathbb{E}[\neg g]$$

Proof that $\Delta \mapsto \sqrt{\Delta}\,\mathrm{polylog}\, n$ (continued)

- $\mathcal{B} = \{h : \mathbb{E}[h] \approx 1 - \rho\}$
- How big can $\mathcal{B}$ be?
- $$1/\,\mathrm{poly}(n) \leq \mathbb{E}[\neg g] \leq \prod_{h \in \mathcal{B}} \mathbb{E}[h]$$

Proof that $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$ (continued)

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- How big can $\mathcal{B}$ be?
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Proof that $\Delta \mapsto \sqrt{\Delta}$ polylog n (continued)

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- So $|\mathcal{B}| \leq O((1/\rho) \log n)$
- We also trivially have $|\mathcal{B}| \leq \Delta$

Proof that $\Delta \mapsto \sqrt{\Delta}\,\text{polylog}\, n$ (continued)

- Let $L = \#\{h \in \mathcal{B} : h|_{X^{\circ s}} \neq 1\}$ (number of living children)

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Proof that $\Delta \mapsto \sqrt{\Delta}\,\mathrm{polylog}\, n$ (continued)

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$$\leq \left(\frac{|\mathcal{B}|e}{M}\right)^k \cdot \left(O(\rho)^k + \frac{1}{n^{200}}\right) \qquad \text{Stirling}$$

Proof that $\Delta \mapsto \sqrt{\Delta}\,\mathrm{polylog}\,n$ (continued)

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- $$\begin{aligned}
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&\leq \left(\frac{|\mathcal{B}|e}{M}\right)^k \cdot \left(O(\rho)^k + \frac{1}{n^{200}}\right) && \text{Stirling}\\
&\leq \left(\frac{1}{\sqrt{\Delta}}\right)^k + \frac{1}{n^{200}} \cdot (\sqrt{\Delta})^k \\
&\leq \frac{2}{n^{100}} && \square
\end{aligned}$$

Finishing proof of main lemma

- After $s = O(\log \log n)$ restrictions, $\Delta \mapsto \sqrt{\Delta} \cdot \text{polylog}\, n$

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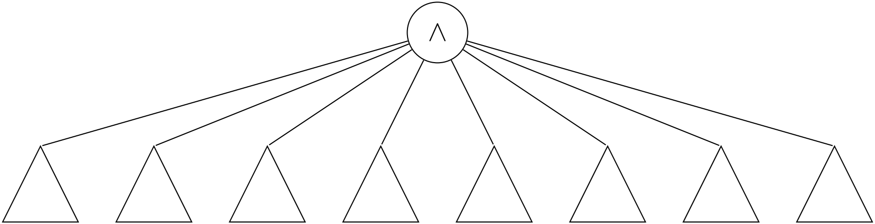
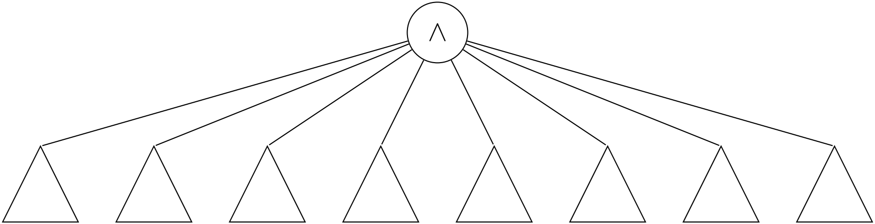


Finishing proof of main lemma

- After $s = O(\log \log n)$ restrictions, $\Delta \mapsto \sqrt{\Delta} \cdot \mathrm{polylog}\, n$
- Therefore, after $t = O((\log \log n)^2)$ restrictions, $\Delta = \mathrm{polylog}\, n$
- Total cost so far: $\widetilde{O}(\log n)$ truly random bits



Final step: MRT PRG

- **Theorem** (Meka, Reingold, Tal '19): There is an explicit PRG with seed length $\widetilde{O}(\log(n/\varepsilon))$ that fools functions of the form

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- In our case,

$$f = \bigwedge_{i=1}^{m} f_i$$

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- **Theorem** (Meka, Reingold, Tal '19): There is an explicit PRG with seed length $\widetilde{O}(\log(n/\varepsilon))$ that fools functions of the form

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where $f_1, \ldots, f_m$ are on disjoint variables and f_i can be computed by an ROBP with width $O(1)$, length $\operatorname{polylog} n$

- (Proof uses GMRTV approach, building on [GY14, CHRT18, Vio09])

- In our case,

$$f = \bigwedge_{i=1}^{m} f_i = \sum_{S \subseteq [m]} \frac{(-1)^{|S|}}{2^m} \prod_{i \in S} (-1)^{f_i}$$

Directions for further research

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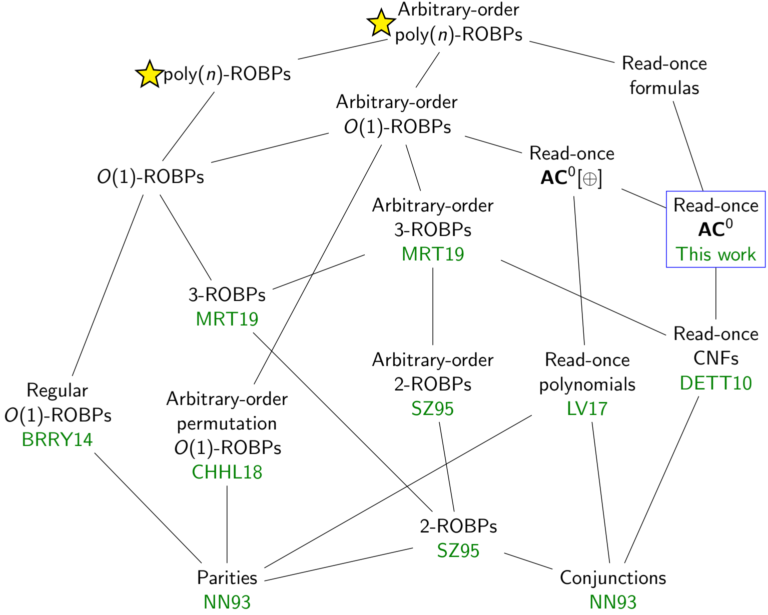

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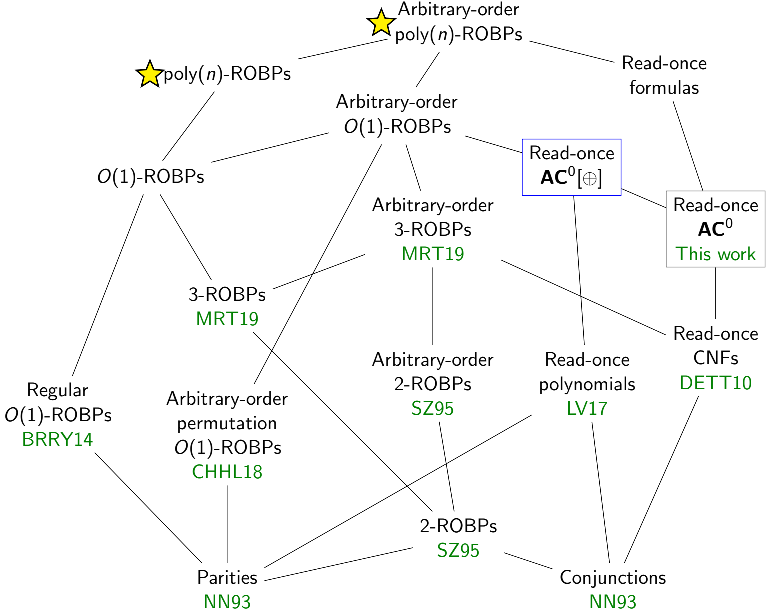
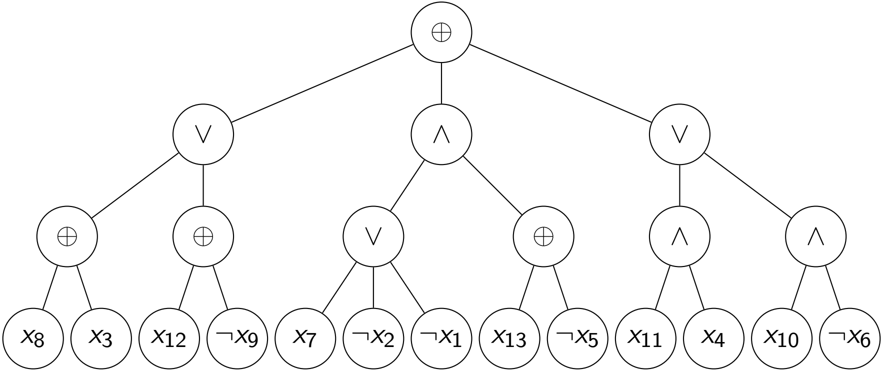


Read-once $\mathbf{AC}^0[\oplus]$



Fooling read-once $\mathbf{AC}^0[\oplus]$

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- Thanks! Questions?